Q-Cut Logistics

Calabrese Giuseppe, Chiurco Mattia,
Mairoll Emanuel, Scarinzi Thomas

The Problem

Exact problem definition (from CMT Dataset):

- Coordinate-located, weighted nodes (“demand”)
- Weighted, undirected edges (“euclidean distance”)
- Graphs are fully connected

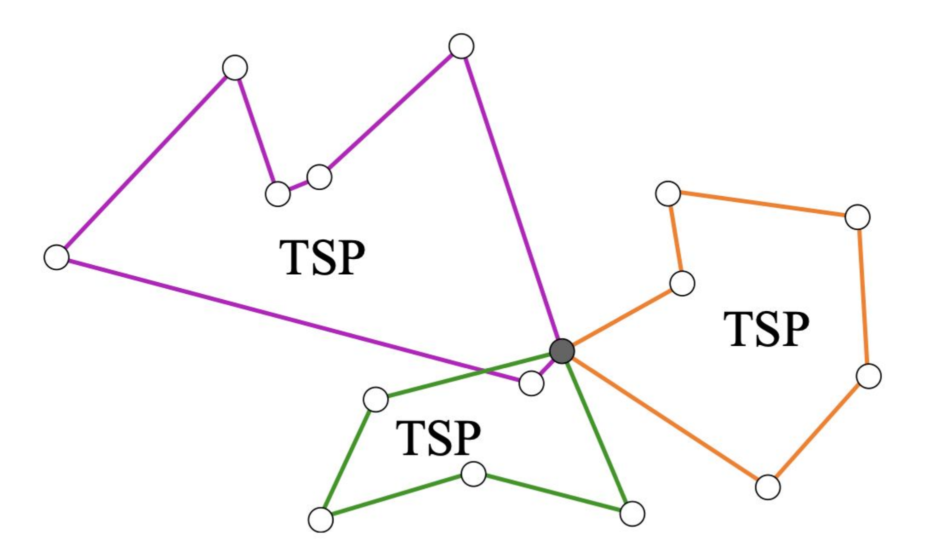
Applicable for all kinds Delivery Services, Supply Chain:

- Drone-based delivery networks
- Waste retrieval and recycling
- Heating circuits in Buildings

Note on the dataset used: Christofides, Mingozzi and Toth (1979)

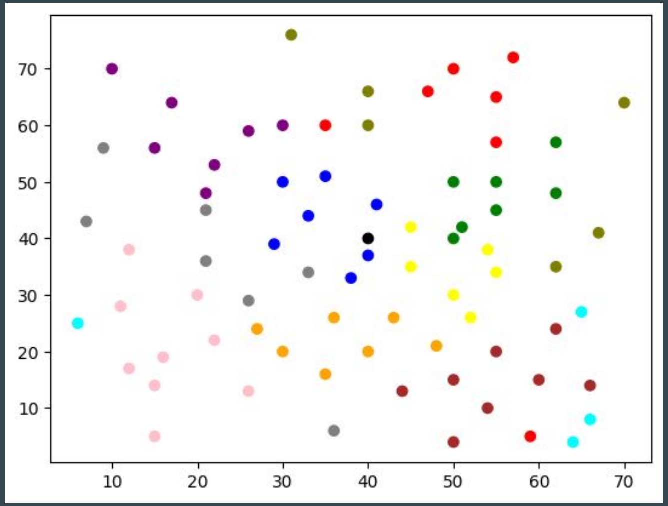
Two-Phase Decomposition of CVRP

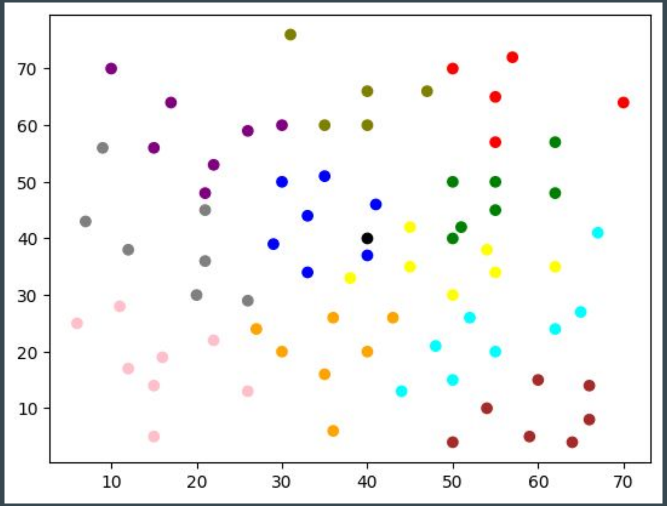
- Principle from Herzog et. al.
- Clustering Phase
  - Breaking up into multiple TSP
- Routing Phase
  - Solving each TSP individually



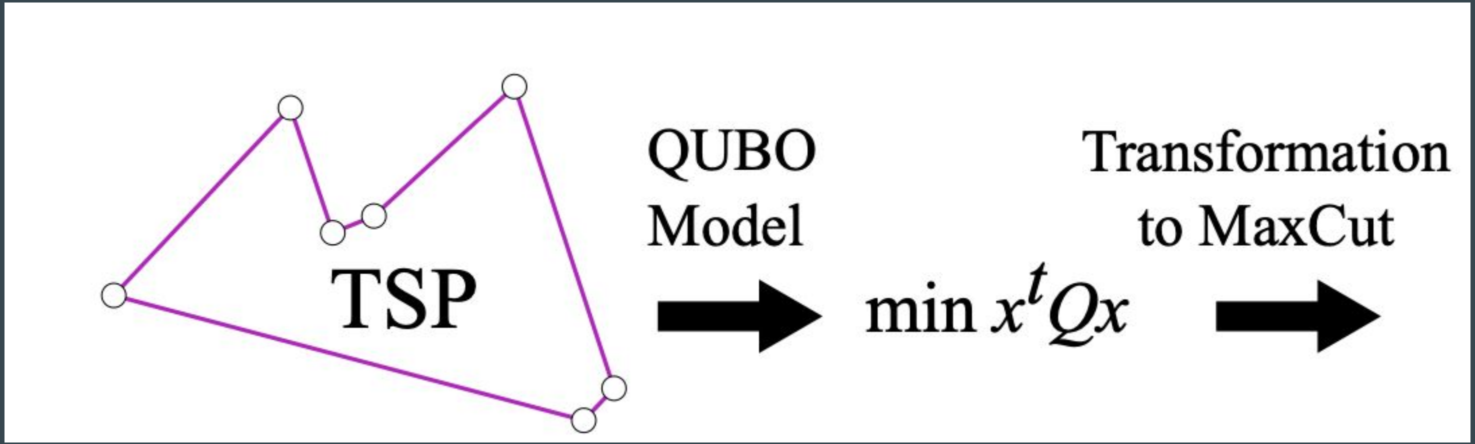
Clustering Phase

- Classical Approach from Feld et. al.
- Also 2 Phases
  - Cluster Generation: Greedy Fill
  - Cluster Improvement: Random Replace





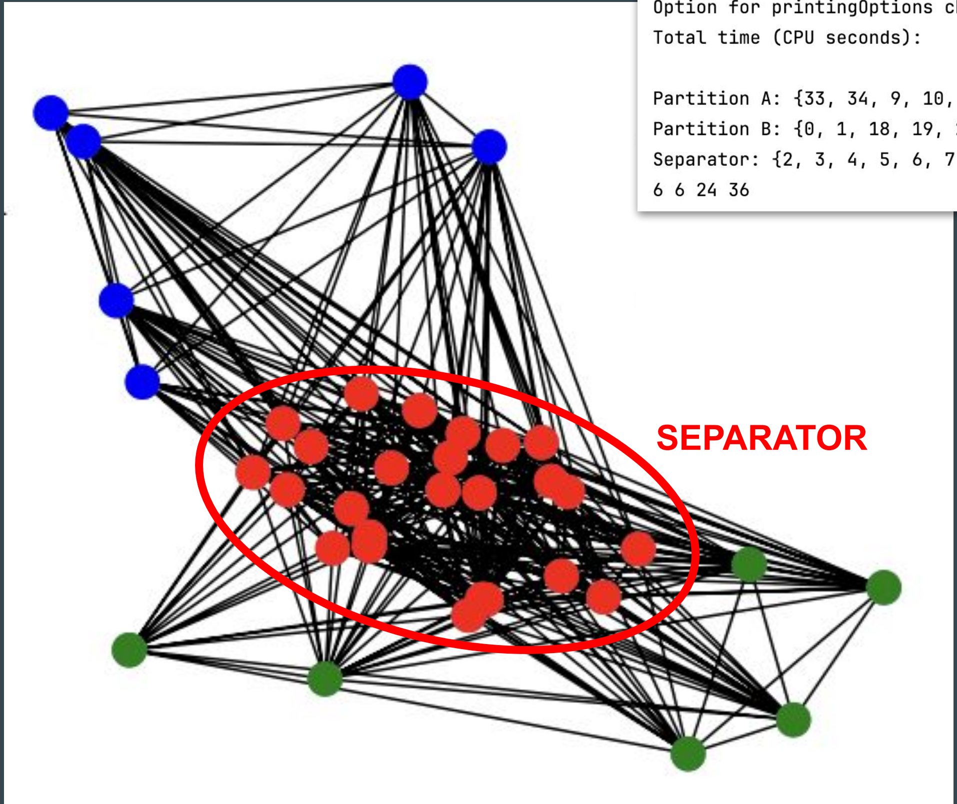
Routing Phase



QUBO	Number of Nodes:	$(n-1)^2$	(since the starting point is fixed)
MaxCut:	Number of Nodes:	$(n-1)^2 + 1$	

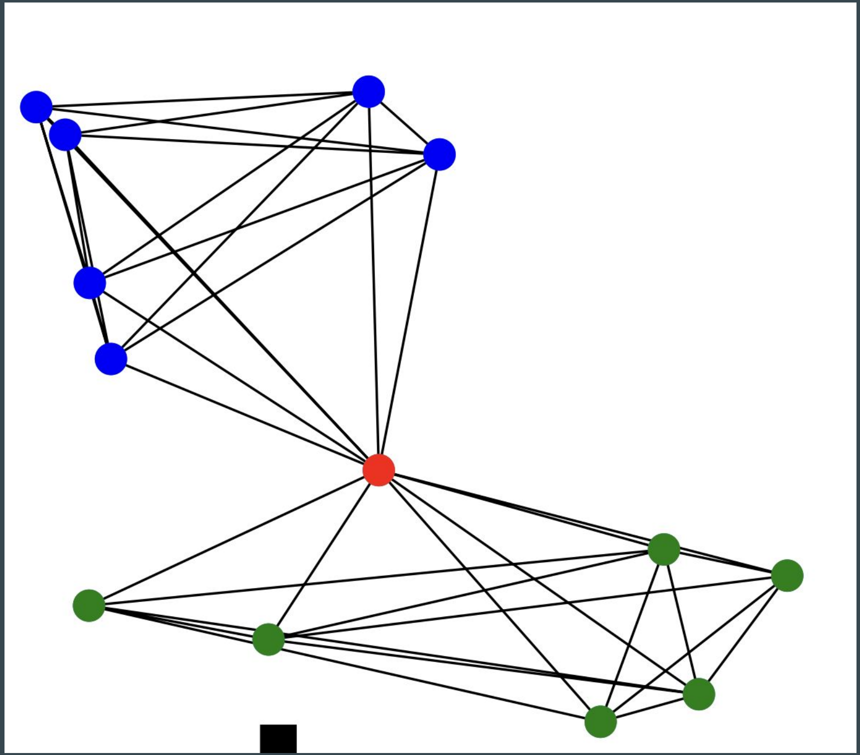
Graph Shrinking

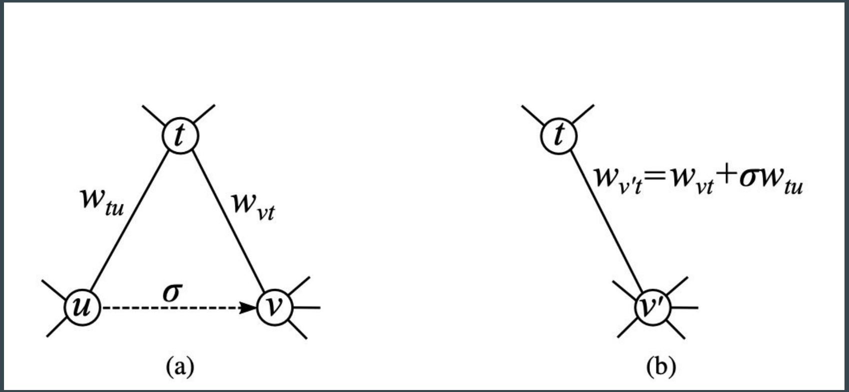
```
Objective value:                12.00000000
Enumerated nodes:               160
Total iterations:               16942
Time (CPU seconds):             4.71
Time (Wallclock seconds):       4.76

Option for printingOptions changed from normal to all
Total time (CPU seconds):       4.72   (Wallclock seconds):       4.76

Partition A: {33, 34, 9, 10, 15, 16}
Partition B: {0, 1, 18, 19, 24, 25}
Separator: {2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 35}
6 6 24 36
```



$$
\begin{aligned}
\max_{x,y} \quad & \sum_{v \in V} x_v + y_v \\
\text{s.t.} \quad & x_u + y_v \leq 1 \quad \forall uv \in E \\
& x_v + y_u \leq 1 \quad \forall uv \in E \\
& x_v + y_v \leq 1 \quad \forall v \in V \\
& \sum_{v \in V} x_v - y_v \leq \beta \\
& -\sum_{v \in V} x_v - y_v \leq \beta \\
& x_v, y_v \in \{0, 1\} \quad \forall v \in V
\end{aligned}
$$

$$
\begin{aligned}
\max_{x} \; & \sum_{e \in E} w_e x_e \\
\text{s.t.} \; & \sum_{e \in T} x_e - \sum_{e \in C \setminus T} x_e \le |T| - 1, \\
& \qquad \forall T \subseteq C,\ |T| \text{ odd},\ \forall C \subseteq E \text{ cycle} \\
& 0 \le x_e \le 1 \quad \forall e \in E \\
& x_e \in \{0,1\} \quad \forall e \in E\ .
\end{aligned}
$$





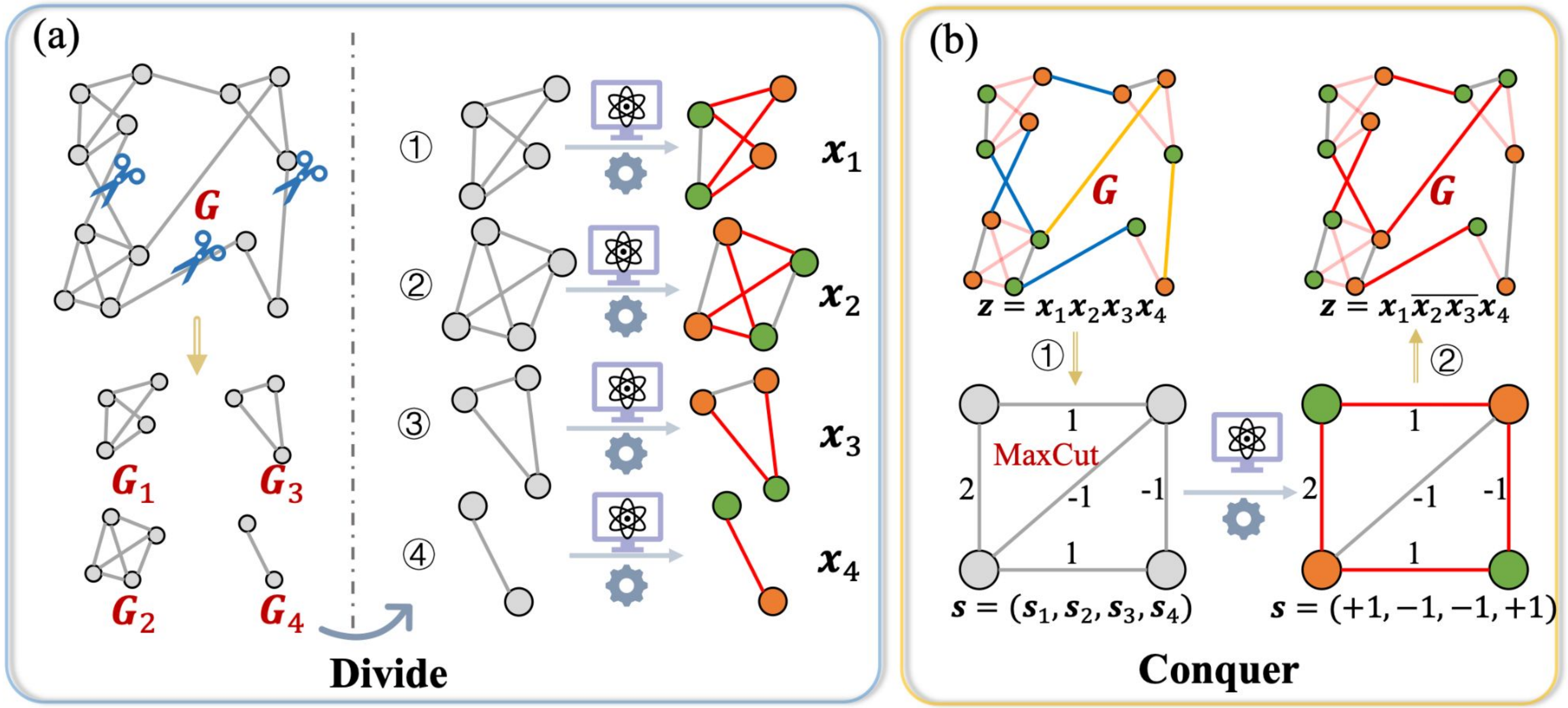
Alternative Approaches

Goemans-Williamson Algorithm:

- Semidefinite programming to find an approximation to the MaxCut problem
- It guarantees an approximation ratio of about 0.87856

Divide and Conquer Strategy:

- Tried to use the GW Algorithm as an heuristic to find partitions of the entire graph
- Apply the QAOA to small partitions we know how to solve
- Merge the solutions and iterate this process



(a)
G
G_1
G_3
G_2
G_4
①
②
③
④
x_1
x_2
x_3
x_4
Divide
(b)
G
G
$z = x_1 x_2 x_3 x_4$
$z = x_1 \overline{x_2 x_3} x_4$
①
②
MaxCut
1
2
-1
-1
1
$s = (s_1, s_2, s_3, s_4)$
$s = (+1, -1, -1, +1)$
Conquer